

Consider the conic with equation  $2x^2 - 3y^2 + 8x + 18y - 1 = 0$ .

SCORE: \_\_\_\_ / 30 PTS

- [a] Find the co-ordinates of the focus/foci.

$$2x^2 + 8x - 3y^2 + 18y = 1$$

$$2(x^2 + 4x + 4) - 3(y^2 - 6y + 9) = 1 + 8 - 27$$

$$2(x+2)^2 - 3(y-3)^2 = -18$$


$$\frac{(y-3)^2}{6} - \frac{(x+2)^2}{9} = 1$$

$$c^2 = 6 + 9 = 15$$

$$(-2, 3 \pm \sqrt{15})$$

- [b] If the equation corresponds to a circle, find its radius.

If the equation corresponds to a parabola, find its vertex & directrix.

If the equation corresponds to an ellipse, find the endpoints of its minor axes.

If the equation corresponds to a hyperbola, find the equations of the asymptotes.

$$y - 3 = \pm \frac{\sqrt{6}}{3}(x + 2)$$

Pat & Reese live in separate houses on Taylor Street.  
Taylor Street is parallel to Jordan Road. Both roadways are straight.

SCORE: \_\_\_\_ / 10 PTS

Draw diagrams and write algebraic equations involving distances to answer the following questions.

REESE'S

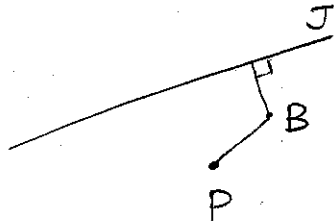
- [a] A bird is flying through the neighborhood. The bird is always 300 meters closer to Pat's house than to ~~Chris's~~ Reese's house.  
What is the shape of the bird's path? (Assume the bird is flying at a constant low height.)



$$BR - BP = 300$$

HYPERBOLA

- [b] There is a bike path nearby. Every point on the path is three times as far from Jordan Road as it is from Pat's house.  
What is the shape of the bike path?



$$BJ = 3 \times BP$$

$$e = \frac{1}{3} = \frac{BP}{BJ}$$

ELLIPSE

Consider the ellipse with foci  $(9, -10)$  and  $(9, 2)$  and minor axis of length 8.

SCORE: \_\_\_\_ / 20 PTS

[a] Find the equation of the ellipse.

$$\text{CENTER} = (9, \frac{-10+2}{2}) = (9, -4)$$

$$2b = 8 \rightarrow b = 4$$

$$c = 2 - (-4) = 6$$

$$a^2 = 4^2 + 6^2 = 52$$

$$\frac{(x-9)^2}{4^2} + \frac{(y+4)^2}{52} = 1 \rightarrow \frac{(x-9)^2}{16} + \frac{(y+4)^2}{52} = 1$$

• V

•  $F(9, 2)$

•  $c(9, -4)$

•  $F(9, -10)$

• V

[b] Find the co-ordinates of the vertices.

$$(9, -4 \pm 2\sqrt{13})$$

Convert the rectangular equation  $x = 3y - 5$  to polar.

SCORE: \_\_\_\_ / 15 PTS

$$r \cos \theta = 3r \sin \theta - 5$$

$$r \cos \theta - 3r \sin \theta = -5$$

$$r(\cos \theta - 3 \sin \theta) = -5$$

$$r = - \frac{5}{\cos \theta - 3 \sin \theta}$$

$$r = \frac{5}{3 \sin \theta - \cos \theta}$$

Consider the polar equation  $r = -5 - 4 \sin \theta$ .

SCORE: \_\_\_\_ / 30 PTS

**The following symmetry tests all fail:  $(-r, \theta)$ ,  $(-r, -\theta)$  and  $(-r, \pi - \theta)$**

- [a] Run the other standard tests for symmetry for the polar equation, and summarize all conclusions in the table below.

$$\theta = \frac{\pi}{2}: (r, \pi - \theta)$$

$$r = -5 - 4 \sin(\pi - \theta)$$

$$r = -5 - 4 [\sin \pi \cos \theta - \cos \pi \sin \theta]$$

$$r = -5 - 4 \sin \theta$$

|                                           |               |
|-------------------------------------------|---------------|
| SYMMETRIC OVER $\theta = \frac{\pi}{2}$ ? | YES           |
| SYMMETRIC OVER POLE?                      | NO CONCLUSION |
| SYMMETRIC OVER POLAR AXIS?                | NO CONCLUSION |

$$\text{POLE: } (r, \pi + \theta)$$

$$r = -5 - 4 \sin(\pi + \theta)$$

$$r = -5 - 4 [\sin \pi \cos \theta + \cos \pi \sin \theta]$$

$$r = -5 + 4 \sin \theta$$

$$\text{POLAR AXIS: } (r, -\theta)$$

$$r = -5 - 4 \sin(-\theta)$$

$$r = -5 + 4 \sin \theta$$

- [b] What is the minimum interval of  $\theta$  - values that must be plotted before using symmetry to complete the graph?

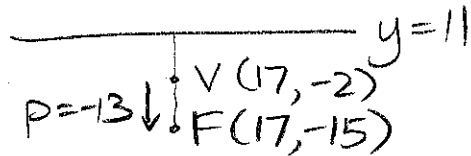
$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

- [c] Name the shape of the graph of the polar equation. If the graph is a rose curve, state the number of petals.

$$|-4| < |-5| < 2|-4| \rightarrow \text{LIMACON WITH DIMPLE}$$

Find the equation of the parabola with focus  $(17, -15)$  and directrix  $y = 11$ .

SCORE: \_\_\_\_ / 15 PTS



$$\text{VERTEX} = \left(17, \frac{-15+11}{2}\right) = (17, -2)$$

$$p = -15 - (-2) = -13$$

$$(x-17)^2 = 4(-13)(y+2)$$

$$(x-17)^2 = -52(y+2)$$

Consider the conic with the polar equation  $r = \frac{32}{9 - 7 \cos \theta}$ .

SCORE: \_\_\_\_ / 30 PTS

- [a] What is the shape of the graph of the equation?

$$r = \frac{\frac{32}{9}}{1 - \frac{7}{9} \cos \theta} \quad e = \frac{7}{9} < 1 \rightarrow \text{ELLIPSE}$$

- [b] Find the equation of the directrix.

$$ep = \frac{32}{9}$$

$$\frac{7}{9}p = \frac{32}{9}$$

$$p = \frac{32}{7}$$

$$x = -\frac{32}{7}$$

- [c] Find the rectangular co-ordinates of the focus/foci, using the process in the lecture & website handout.

| $\theta$         | $r$            | $(x, y)$             |
|------------------|----------------|----------------------|
| 0                | 16             | (16, 0)              |
| $\frac{\pi}{2}$  | $\frac{32}{9}$ | $(0, \frac{32}{9})$  |
| $\pi$            | 2              | (-2, 0)              |
| $\frac{3\pi}{2}$ | $\frac{32}{9}$ | $(0, -\frac{32}{9})$ |

VERTICES  $\leftarrow$

CENTER =  $\left(\frac{16-2}{2}, 0\right)$   
 $= (7, 0)$

FOCI =  $(14, 0),$   
 $(0, 0)$